Short Tricks - JEE-Main

By

Ajay Singh Jadon (A.S. Sir)

(IIT-MATHEMATICS)

(Ex-9yr, Author of "DOOR TO IIT")

1. Progression

Lecture - 1

Q.1 If the ratio of sum of n terms of two A.P's is $(3n + 8) : (7n + 15)$, then the ratio of 12^{th} terms is-

(A) 16 : 7 (B) 7 :16 (C) 7 : 12 (D) 12 : 5

Sol. [B]

Proper Method	Short Trick
$\frac{S_n}{S'_n} = \frac{3n+8}{7n+15}$ $\Rightarrow \frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]} = \frac{3n+8}{7n+15}$ $\Rightarrow \frac{a_1+\left(\frac{n-1}{2}\right)d_1}{a_2+\left(\frac{n-1}{2}\right)d_2} = \frac{3n+8}{7n+15}$ put $\frac{n-1}{2} = 11 \therefore n = 23$ $\Rightarrow \frac{a_1+11d_1}{a_2+11d_2} = \frac{3.23+8}{7.23+15} = \frac{77}{176} = \frac{7}{16}$	$\frac{S_n}{S'_n} = \frac{3n+8}{7n+15}$ Then $\frac{T_n}{T'_n} = \frac{3(2n-1)+8}{7(2n-1)+15}$ $\frac{T_{12}}{T'_{12}} = \frac{7}{16}$

Q.2 If the ratio of the sum of n terms of two AP's is $2n : (n + 1)$, then ratio of their 8^{th} terms is-

(A) 15 : 8   (B) 8 : 13   (C) $n : (n - 1)$ (D) 5 : 17

Sol. **[A]**

Proper Method

$$\frac{S_n}{S_n'} = \frac{2n}{n+1}$$

$$\Rightarrow \frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{2n}{n+1}$$

$$\Rightarrow \frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{2n}{n+1}$$

put $\frac{n-1}{2} = 7 \therefore n = 15$

$$\Rightarrow \frac{a_1 + 7d_1}{a_2 + 7d_2} = \frac{2(15)}{15+1} = \frac{15}{8}$$

Short Trick

$$\frac{S_n}{S'_n} = \frac{2n}{n+1}$$

$$\Rightarrow \frac{T_n}{T'_n} = \frac{2(2n-1)}{(2n-1)+1}$$

$$\frac{T_8}{T'_8} = \frac{15}{8}$$

Q.3 The sum to infinity of the following series $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+....$ shall be-

(A) ∞ (B) 1 (C) 0 (D) None of these

Sol. [B]

Proper Method	Short Trick
$S_n = \frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}\ \ldots\ldots\ldots + \frac{1}{n(n+1)}$ $S_n = \left(\frac{1}{1}-\frac{1}{2}\right) + \left(\frac{2}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) \ldots\ldots\ldots + \left(\frac{1}{n}-\frac{1}{n+1}\right)$ $S_n = 1-\frac{1}{n+1} = \frac{n}{n+1}$ $S_\infty = \lim_{n\to\infty}\left(\frac{n}{n+1}\right) = 1$	$\text{Ans} = \frac{1}{(1)(2-1)} = 1$

Q.4 $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+$....... upto ∞ terms is-

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{1}{3}$ (D) None

Sol. [C]

Proper Method	Short Trick
$S_n = \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + ... + \frac{1}{(3n-2)(3n+1)}$ $S_n = \frac{1}{3}\left[\frac{4-1}{1.4} + \frac{7-4}{4.7} + ... + \frac{(3n+1)-(3n-2)}{(3n+1)(3n-2)}\right]$ $S_n = \frac{1}{3}\left[\left(\frac{1}{1} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{10}\right) + ... + \left(\frac{1}{3n-2} - \frac{1}{3n+1}\right)\right]$ $S_n = \frac{1}{3}\left[1 - \frac{1}{3n+1}\right] \Rightarrow S_\infty = \lim_{n\to\infty} \frac{1}{3}\left[1 - \frac{1}{3n+1}\right]$ $= \frac{1}{3}$	$\text{Ans} = \frac{1}{1(4-1)} = \frac{1}{3}$

Q.5 If A.M. between p and q ($p \geq q$) is two times the GM, then p : q is-

(A) 1 : 1 (B) 2 : 1 (C) $(2 + \sqrt{3}) : (2 - \sqrt{3})$ (D) 3 :1

Sol. [C]

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<tr><th>Proper Method</th><th>Short Trick</th></tr>
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Let AM of p and q is A and G.M of p and q is G then

$A = \frac{p+q}{2}$, $G = \sqrt{pq}$

Given that $\frac{A}{G} = \frac{2}{1}$

$\Rightarrow \frac{p+q}{2\sqrt{pq}} = \frac{2}{1}$

Using componendo & dividendo

$\Rightarrow \frac{p+q+2\sqrt{pq}}{p+q-2\sqrt{pq}} = \frac{2+1}{2-1}$

$\Rightarrow \left(\frac{\sqrt{p}+\sqrt{q}}{\sqrt{p}-\sqrt{q}}\right)^2 = \frac{3}{1}$

$\Rightarrow \frac{\sqrt{p}+\sqrt{q}}{\sqrt{p}-\sqrt{q}} = \frac{\sqrt{3}}{1}$

$\Rightarrow$ again using componendo & dividendo

$\Rightarrow \frac{(\sqrt{p}+\sqrt{q})+(\sqrt{p}-\sqrt{q})}{(\sqrt{p}+\sqrt{q})-(\sqrt{p}-\sqrt{q})} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$\Rightarrow \frac{\sqrt{p}}{\sqrt{q}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$\Rightarrow \frac{p}{q} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$

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$\frac{A.M}{G.M} = \frac{2}{1}$

$\frac{p}{q} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$

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Q.6 If the arithmetic mean of two numbers a, b ; a > b > 0 is five times their geometric mean, then $\frac{a+b}{a-b}$ is equal to –

(A) $\frac{7\sqrt{3}}{12}$ (B) $\frac{3\sqrt{2}}{4}$ (C) $\frac{\sqrt{6}}{2}$ (D) $\frac{5\sqrt{6}}{12}$

Sol. [D]

Proper Method

Let AM of a and b is A and a.m of a and b is G then

$A = \frac{a+b}{2}$, $G = \sqrt{ab}$

Given that $\frac{A}{G} = \frac{5}{1}$

$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{5}{1}$

Using compounds & dividends

$\Rightarrow \frac{a+b+\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{5+1}{5-1}$

$\Rightarrow \left(\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right)^2 = \frac{6}{4}$

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{6}}{2}$

$\Rightarrow$ again using compounds & dividends

$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})+(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})-(\sqrt{a}-\sqrt{b})} = \frac{\sqrt{6}+2}{\sqrt{6}-2}$

$\Rightarrow \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{6}+2}{\sqrt{6}-2}$

$\Rightarrow \frac{a}{b} = \frac{12+4\sqrt{6}}{12-4\sqrt{6}}$

$\Rightarrow \frac{a+b}{a-b} = \frac{12}{8\sqrt{6}} = \frac{5\sqrt{6}}{12}$

Short Trick

$\frac{A.M}{G.M} = \frac{5}{1}$

$\frac{a}{b} = \frac{5+\sqrt{24}}{5-\sqrt{24}}$

$\frac{a+b}{a-b} = \frac{5\sqrt{6}}{12}$

Q.7 The fractional value of $0.\dot{1}\dot{2}\dot{5}$ is-

(A) 125/999 (B) 23/990 (C) 61/550 (D) None of these

Sol. [A]

Proper Method	**Short Trick**
Let $x = 0.\overline{125}$ $\therefore\ x = 0.125125125$ $\therefore\ 1000x = 125.125125125$ Now (ii) – (i); $999x = 125$ $\therefore\ x = 125/999$	$0.\dot{1}\dot{2}\dot{5} = \dfrac{125-0}{999} = \dfrac{125}{999}$